Determine if each series below is convergent or divergent. **Do NOT find the sum.** SCORE: _____ / 16 PTS

[a] $\displaystyle\sum_{n=1}^{\infty}\frac{1}{1+e^{-n}}$ SUBTOTAL = 3

1 $\displaystyle\lim_{n\to\infty}\frac{1}{1+e^{-n}}=\frac{1}{1+0}=1\neq 0$ 1

SERIES DIVERGES

1

[b] $\displaystyle\sum_{n=1}^{\infty}\frac{\sqrt{n+2}}{2n^2+n+1}$ SUBTOTAL = 8

COMPARE TO $\displaystyle\sum_{n=1}^{\infty}\frac{\sqrt{n}}{2n^2}=\frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{n^{\frac{3}{2}}}$

2 $\displaystyle\lim_{n\to\infty}\frac{\frac{\sqrt{n+2}}{2n^2+n+1}}{\frac{1}{2n^{\frac{3}{2}}}}$

$\displaystyle=\lim_{n\to\infty}\frac{2n^{\frac{3}{2}}\sqrt{n+2}}{2n^2+n+1}\cdot\frac{\frac{1}{n^2}}{\frac{1}{n^2}}$

1 $\displaystyle=\lim_{n\to\infty}\frac{2\sqrt{1+\frac{2}{n}}}{2+\frac{1}{n}+\frac{1}{n^2}}=\frac{2\sqrt{1+0}}{2+0+0}=1\neq 0$ 1 1

1 $\displaystyle\frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{n^{\frac{3}{2}}}$ CONVERGES $(p=\frac{3}{2}>1)$ 1

SO $\displaystyle\sum_{n=1}^{\infty}\frac{\sqrt{n+2}}{2n^2+n+1}$ CONVERGES

1

SUBTOTAL = 5

[c] $\displaystyle\sum_{n=1}^{\infty}\frac{1}{ne^n}$ 1

$\displaystyle 0<\frac{1}{ne^n}\leq\frac{1}{e^n}$ FOR $n\geq 1$ 1

1 $\displaystyle\sum_{n=1}^{\infty}\frac{1}{e^n}$ CONVERGES $(r=\frac{1}{e}<1)$ 1

SO $\displaystyle\sum_{n=1}^{\infty}\frac{1}{ne^n}$ CONVERGES

1

Consider the following statements. **SCORE: _____ / 3 PTS**

(i) If $a_n \le b_n$ and $\sum_{n=1}^{\infty} a_n$ is divergent, then $\sum_{n=1}^{\infty} b_n$ is divergent

(ii) If the terms of $\{a_n\}$ and $\{b_n\}$ are positive, and $\lim_{n\to\infty} \frac{a_n}{b_n} \ne 0$, and $\sum_{n=1}^{\infty} b_n$ is convergent, then $\sum_{n=1}^{\infty} a_n$ is convergent

(iii) If $0 \le b_n \le a_n$ and $\sum_{n=1}^{\infty} a_n$ is convergent, then $\sum_{n=1}^{\infty} b_n$ is convergent

Which of the statements above are true ? Circle the correct answer below.

[a]	none are true	[b]	only (i) is true	[c]	only (ii) is true	[d]	only (iii) is true
[e]	only (i) and (ii) are true	[f]	only (i) and (iii) are true	[g]	only (ii) and (iii) are true	[h]	all are true

It is true that $\int_1^\infty e^{-\pi x}\cos \pi x\, dx$ converges. A student writes that this means that $\sum_{n=1}^\infty e^{-\pi n}\cos \pi n$ converges according to the integral test. Is the student's reasoning correct ? Why or why not ?

SCORE: _____ / 2 PTS

$\frac{1}{2}$ INCORRECT, $e^{-\pi x}\cos \pi x$ OSCILLATES (INCREASES + DECREASES)

AND IS NOT ALWAYS POSITIVE (WHEN n IS ODD)

$1\frac{1}{2}$ IF YOU GOT EITHER ANSWER/REASON

Determine if $\sum_{n=1}^\infty \frac{e^{\frac{1}{n}}}{n^2}$ is convergent or divergent, **without using the comparison and limit comparison tests.**

SCORE: _____ / 9 PTS

$f(x) = \frac{e^{\frac{1}{x}}}{x^2} > 0$ $\frac{1}{2}$, CONTINUOUS $\frac{1}{2}$, ON $[1,\infty)$

$$f'(x) = \frac{e^{\frac{1}{x}}\cdot\left(-\frac{1}{x^2}\right)x^2 - 2xe^{\frac{1}{x}}}{(x^2)^2} = -\frac{e^{\frac{1}{x}}(1+2x)}{x^4} \; (2) \; < 0 \; \left(\tfrac{1}{2}\right), \text{ ON } [1,\infty)$$

SO $\frac{1}{2}$ $f(x)$ IS DECREASING, ON $[1,\infty)$

$$\int \frac{e^{\frac{1}{x}}}{x^2}\,dx = \int -e^u\,du = -e^u + C = -e^{\frac{1}{x}} + C$$

$u = \frac{1}{x}$

$du = -\frac{1}{x^2}dx$

$$\int_1^\infty \frac{e^{\frac{1}{x}}}{x^2}\,dx = \lim_{N\to\infty} \overset{(2)}{-e^{\frac{1}{x}}}\Big|_1^N = \lim_{N\to\infty}\left(-\overset{(1)}{e^{\frac{1}{N}}} + e\right) = -e^0 + e = e - 1$$

CONVERGES (1)

SO $\sum_{n=1}^\infty \frac{e^{\frac{1}{n}}}{n^2}$ CONVERGES (1)